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Quantum statistical properties of the Jaynes–Cummings model in the presence of a classical homogeneous gravitational field

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Abstract

The temporal evolution of quantum statistical properties of an interacting atom–radiation field system in the presence of a classical homogeneous gravitational field is investigated within the framework of the Jaynes–Cummings model. To analyse the dynamical evolution of the atom–radiation system a quantum treatment of the internal and external dynamics of the atom is presented based on an alternative su(2) dynamical algebraic structure. By solving the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the gravitational field on the dynamical behaviour of the atom–radiation system is explored. Assuming that initially the radiation field is prepared in a coherent state and the two-level atom is in a coherent superposition of the excited and ground states, the influence of gravity on the collapses and revivals of the atomic population inversion, atomic dipole squeezing, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field is studied.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The interaction between a two-level atom and a single quantized mode of the electromagnetic field in a lossless cavity within the rotating wave approximation (RWA) can be described by the Jaynes–Cummings model (JCM) [1]. Despite being simple enough to be analytically soluble in the RWA, this model has been a long-lasting source of insight into the nuances of the interaction between light and matter. The JCM has been applied to investigate many quantum effects such as the quantum collapses and revivals of atomic inversion [2], squeezing of the radiation field [3], atomic dipole squeezing [4], vacuum Rabi oscillation [5] and the dynamical entangling and disentangling of the atom–field system in the course of time [6–8]. Investigations of the dynamical behaviour of the JCM are also extremely important due to its experimental realizations in high-Q microwave cavities [9], in optical resonators [10], in laser-cooled trapped ions [11] and in quantum nondemolition measurements [12]. Stimulated by the success of the JCM, more and more people have paid special attention to extending and generalizing the model in order to explore new quantum effects. Discussions related to several interesting generalizations of this model are now available in the literature [13] and the model is still promising in many applications, particularly in the fast developing research area of quantum information [14].

A very significant and noteworthy generalization of the JCM is to include the effect of atomic motion so that the spatial mode structure could be incorporated into this model. In the standard JCM, the interaction between a constant electric field and a stationary (motionless) two-level atom is considered. With the development in the technologies of laser cooling and atom trapping the interaction between a moving atom and the field has attracted much attention [15–24]. In particular, it has been shown that the atomic motion can bring about the nonlinear transient effects similar to self-induced transparency (SIT) and adiabatic following (AF) [25]; the possibility of realizing an optical switching [24] changes the creating time of Schrödinger cat states [21] and exhibits long time entropy squeezing effect [23].

On the other hand, experimentally, atomic beams with very low velocities are generated in laser cooling and atomic interferometry [26]. It is obvious that for atoms moving with a velocity of a few millimetres or centimetres per second for a time period of several milliseconds or more, the influence of Earth's acceleration becomes important and cannot be neglected [27]. For this reason, it is of interest to study the temporal evolution of a moving atom simultaneously exposed to the gravitational field and a single-mode travelling wave field. Since any quantum optical experiment in the laboratory is actually made in a non-inertial frame it is important to estimate the influence of Earth's acceleration on the outcome of the experiment. To get a clear picture of what is going to happen it may be useful to refer to the equivalence principle. It states that the influence of a homogeneous gravitational field on the atom moving in a radiation field can be simulated by constant acceleration. This means that the following situation is physically equivalent to the atom–radiation system exposed to a gravity field: an atom is at rest or moving with constant velocity relative to an inertial system. The laboratory with the radiation field attached to it moves with a constant acceleration. The consequence is that the radiation field reaches the atom with Doppler shifted frequency. Because of the acceleration this shift changes in time. It acts as a time-dependent detuning. A semiclassical description of a two-level atom interacting with a classical running laser wave in a gravitational field has been studied [28, 29]. However, the semiclassical treatment does not permit us to study the pure quantum effects occurring in the course of atom–radiation interaction. Recently, within a quantum treatment of the internal and external dynamics of the atom, we have presented [30] a theoretical scheme based on an su(2) dynamical algebraic structure to investigate the influence of a classical homogeneous gravity field on the quantum nondemolition measurement of atomic momentum in the dispersive JCM.

In this paper we adopt a dynamical algebraic approach to investigate the temporal evolution of quantum statistical properties of the JCM in the presence of a classical homogeneous gravitational field. In the Jaynes–Cummings model, when the atomic motion is in a propagating light wave, we consider a two-level atom interacting with the quantized cavity field in the presence of a homogeneous gravitational field. By solving the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the gravitational field on the dynamical behaviour of the atom–field system is explored. In section 2, we present a quantum treatment of the internal and external dynamics of the atom with an alternative su(2) dynamical algebraic structure within the system. Based on this su(2) structure and in the interaction picture, we obtain an effective Hamiltonian describing the atom–field interaction in the presence of a classical gravity field. In section 3, we investigate the dynamical evolution of the system and show that how the gravitational field may affect the dynamical properties of the JCM. In section 4, we study the influence of gravitational field on both the cavity field and the atomic properties. Considering the field to be initially in a coherent state and the two-level atom in a coherent superposition of the ground and excited states, we investigate the temporal evolution of the atomic inversion, atomic dipole squeezing, atomic momentum diffusion, probability distribution of the cavity field, photon counting statistics and quadrature squeezing of the radiation field. Finally, we summarize our conclusions in section 5.

2. Jaynes–Cummings model in the presence of a gravitational field

The system we consider here is a moving two-level atom of mass M exposed simultaneously to a single-mode travelling wave field and a classical homogeneous gravitational field. Therefore, the Hamiltonian of the atom–field system in the presence of a gravitational field with the atomic motion along the position vector $\hat{\vec{x}}$ and in the RWA is given by

$$\hat{H} = \frac{\hat{p}^2}{2M} - M\vec{g}\cdot\hat{\vec{x}} + \hbar\omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_{eg}\hat{\sigma}_z + \hbar\lambda[\exp(-\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{a}^\dagger\hat{\sigma}_- + \exp(\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{\sigma}_+\hat{a}], \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of a single-mode travelling wave with frequency ω_c, $\vec{q}$ is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $|e\rangle$, $|g\rangle$ and Bohr transition frequency ω_{eg}. The atom–field coupling is given by the parameter λ and $\hat{\vec{p}}$, $\hat{\vec{x}}$ denote, respectively, the momentum and position operators of the atomic centre of mass motion and g is Earth's gravitational acceleration. It has been shown [30] that based on an su(2) algebraic structure, as the dynamical symmetry group of the model, the Hamiltonian (1) can be transformed to the following effective Hamiltonian,

$$\hat{\hat{H}} = \frac{\hat{p}^2}{2M} - \hbar\hat{\triangle}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \frac{1}{2}Mg^2t^2 + \hat{\vec{p}}\cdot\vec{g}t + \hbar(\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+), \tag{2}$$

where $\hat{\kappa}(t)$ is an effective coupling coefficient

$$\hat{\kappa}(t) = \lambda\exp\left(\frac{\mathrm{i}t}{2}\left(\hat{\triangle}(\hat{\vec{p}},\vec{g}) + \frac{\hbar q^2}{M}\right)\right). \tag{3}$$

The operators

$$\hat{S}_0 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|), \qquad \hat{S}_+ = \hat{a}|e\rangle\langle g|\frac{1}{\sqrt{\hat{K}}}, \qquad \hat{S}_- = \frac{1}{\sqrt{\hat{K}}}|g\rangle\langle e|\hat{a}^\dagger, \tag{4}$$

with the following commutation relations,

$$[\hat{S}_0, \hat{S}_\pm] = \pm\hat{S}_\pm, \qquad [\hat{S}_-, \hat{S}_+] = -2\hat{S}_0, \tag{5}$$

are the generators of the su(2) algebra, the operator $\hat{K} = \hat{a}^\dagger\hat{a} + |e\rangle\langle e|$ is a constant of motion which represents the total number of excitations of the atom–radiation system, and the operator

$$\hat{\triangle}(\hat{\vec{p}},\vec{g}) = \omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\hat{\vec{p}}}{M} + \vec{q}\cdot\vec{g}t + \frac{\hbar q^2}{2M}\right), \tag{6}$$

has been introduced as the Doppler shift detuning at time t [30]. The Hamiltonian (2) has the form of the Hamiltonian of the JCM, the only modification being the dependence of the detuning on the conjugate momentum and the gravitational field. In the interaction picture the effective Hamiltonian (2) takes the following form,

$$\hat{\hat{H}}_{\mathrm{int}} = \exp\left(\frac{-\mathrm{i}\hat{\hat{H}}_0 t}{\hbar}\right)\hat{\hat{H}}_I \exp\left(\frac{\mathrm{i}\hat{\hat{H}}_0 t}{\hbar}\right), \tag{7}$$

where

$$\hat{\hat{H}}_0 = -\hbar\hat{\Delta}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \hat{H}(\hat{\vec{p}},\vec{g}), \tag{8}$$

and

$$\hat{\hat{H}}_I = \hbar(\kappa\sqrt{\hat{K}}\hat{S}_- + \kappa^*\sqrt{\hat{K}}\hat{S}_+), \tag{9}$$

with

$$\hat{H}(\hat{\vec{p}},\vec{g}) = \frac{\hat{p}^2}{2M} + \hat{\vec{p}}\cdot\vec{g}t + \frac{1}{2}Mg^2t^2. \tag{10}$$

Therefore we obtain

$$\hat{\hat{H}}_{\mathrm{int}} = \hbar(\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- \exp(-\mathrm{i}t\hat{\Delta}(\hat{\vec{p}},\vec{g})) + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+ \exp(\mathrm{i}t\hat{\Delta}(\hat{\vec{p}},\vec{g}))). \tag{11}$$

Finally by using equation (3) we arrive at

$$\hat{\hat{H}}_{\mathrm{int}} = \hbar\lambda(\sqrt{\hat{K}}\hat{S}_- \exp(-\mathrm{i}t\hat{\Delta}_1(\hat{\vec{p}},\vec{g},t)) + \sqrt{\hat{K}}\hat{S}_+ \exp(\mathrm{i}t\hat{\Delta}_1(\hat{\vec{p}},\vec{g},t))). \tag{12}$$

where

$$\hat{\Delta}_1(\hat{\vec{p}},\vec{g},t) = \frac{1}{2}\left(\omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\hat{\vec{p}}}{M} + \vec{q}\cdot\vec{g}t + 3\frac{\hbar q^2}{2M}\right)\right), \tag{13}$$

is the detuning of the atom–field interaction which depends on both the atomic momentum and the gravitational field.

3. Dynamical evolution

In section 2, we obtained an effective Hamiltonian for the atom–field system in the presence of a classical homogeneous gravitational field in the interaction picture. In this section, we investigate dynamical evolution of the system. We will show how the gravitational field may affect the quantum dynamics of the JCM. For this purpose, we solve the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{\hat{H}}_{\mathrm{int}}|\psi(t)\rangle, \tag{14}$$

for the state vector $|\psi(t)\rangle$ with Hamiltonian (12). Indeed, the two-level atom with momentum $|\vec{p}\rangle$ in the excited state $|e\rangle$ gets annihilated and creates a field excitation. Therefore, the Hamiltonian $\hat{\hat{H}}_{\mathrm{int}}$ transforms the state $|e\rangle\otimes|n\rangle\otimes|\vec{p}\rangle \equiv |e,n\rangle\otimes|\vec{p}\rangle$, where $|n\rangle$ denotes the nth Fock state of the field, into

$$\hat{\hat{H}}_{\mathrm{int}}|e,n\rangle\otimes|\vec{p}\rangle = \hbar\lambda\sqrt{n+1}\exp(-\mathrm{i}t\hat{\Delta}_1(\vec{p},\vec{g},t))|g,n+1\rangle\otimes|\vec{p}\rangle, \tag{15}$$

in which we have used the relations

$$\sqrt{\hat{K}}\hat{S}_-|e,n\rangle = \sqrt{n+1}|g,n+1\rangle, \qquad \hat{\vec{p}}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle. \tag{16}$$

Similarly, the atom with momentum $|\vec{p}\rangle$ in the ground state $|g\rangle$ gets excited at the expense of annihilation a field excitation. Hence, the Hamiltonian transforms the state $|g\rangle\otimes|n+1\rangle\otimes|\vec{p}\rangle \equiv |g,n+1\rangle\otimes|\vec{p}\rangle$ into

$$\hat{\hat{H}}_{\mathrm{int}}|g,n+1\rangle\otimes|\vec{p}\rangle = \hbar\lambda\sqrt{n+1}\exp(\mathrm{i}t\hat{\Delta}_1(\vec{p},\vec{g},t))|e,n\rangle\otimes|\vec{p}\rangle. \tag{17}$$

Since the Hamiltonian couples only the states $|g, n+1\rangle \otimes |\vec{p}\rangle$ and $|e, n\rangle \otimes |\vec{p}\rangle$ we introduce the state vector

$$|\psi(t)\rangle = \int \mathrm{d}^3 p \sum_{n=0} (\psi_{e,n}(\vec{p}, \vec{g}, t)|e, n\rangle \otimes |\vec{p}\rangle + \psi_{g,n+1}(\vec{p}, \vec{g}, t)|g, n+1\rangle \otimes |\vec{p}\rangle)$$
$$+ \int \mathrm{d}^3 p \psi_{g,0}(\vec{p}, t)|g, 0\rangle \otimes |\vec{p}\rangle. \tag{18}$$

The state $|g, 0\rangle$, which corresponds to $n = -1$ in equation (17), plays a special role. According to equation (17) we find $\hat{\tilde{H}}_{\mathrm{int}}|g, 0\rangle = 0$ which means, the vacuum cannot excite an atom initially in the ground state and therefore, the state $|g, 0\rangle$ decouples from the rest of the states.

Now we find the equations of motion for the time-dependent probability amplitudes $\psi_{e,n}(\vec{p}, \vec{g}, t) \equiv \psi_1$, $\psi_{g,n+1}(\vec{p}, \vec{g}, t) \equiv \psi_2$ by substituting (18) into (14) and making use of equations (15) and (17)

$$\dot{\psi}_1 = -\mathrm{i}\lambda\sqrt{n+1}\exp(\mathrm{i}\triangle_1(\vec{p}, \vec{g}, t)t)\psi_2, \tag{19}$$

and

$$\dot{\psi}_2 = -\mathrm{i}\lambda\sqrt{n+1}\exp(-\mathrm{i}\triangle_1(\vec{p}, \vec{g}, t)t)\psi_1. \tag{20}$$

At time $t = 0$ the atom is uncorrelated with the field and the state vector of the system can be written as a direct product

$$|\psi(t=0)\rangle = |\psi_{c.m}(0)\rangle \otimes |\psi_{\mathrm{atom}}(0)\rangle \otimes |\psi_{\mathrm{field}}(0)\rangle$$
$$= \left(\int \mathrm{d}^3 p\phi(\vec{p})|\vec{p}\rangle\right) \otimes (c_e|e\rangle + c_g|g\rangle) \otimes \left(\sum_{n=0} w_n|n\rangle\right), \tag{21}$$

where we have assumed that initially the field is in a coherent superposition of Fock states, the atom is in a coherent superposition of its excited and ground states, and the state vector for the centre-of-mass degree of freedom is $|\psi_{c.m}(0)\rangle = \int \mathrm{d}^3 p\phi(\vec{p})|\vec{p}\rangle$. In notation (17) the initial state (21) reads

$$|\psi(t=0)\rangle = \int \mathrm{d}^3 p \sum_{n=0} (w_n c_e \phi(\vec{p})|e, n\rangle \otimes |\vec{p}\rangle + w_{n+1} c_g \phi(\vec{p})|g, n+1\rangle \otimes |\vec{p}\rangle)$$
$$+ \int \mathrm{d}^3 p w_0 \phi(\vec{p}) c_g |g, 0\rangle \otimes |\vec{p}\rangle. \tag{22}$$

When we compare (22) with (18) we find the following initial conditions:

$$\psi_1(t=0) = w_n c_e \phi(\vec{p}), \qquad \psi_2(t=0) = w_{n+1} c_g \phi(\vec{p}), \qquad \psi_{g,0}(t=0) = w_0 c_g \phi(\vec{p}). \tag{23}$$

We can solve two coupled first-order differential equations (19) and (20) in a straightforward way. We have

$$\frac{\partial^2 \psi_1}{\partial t^2} + 2\mathrm{i}\vec{q}\cdot\vec{g}\left(t - \frac{\triangle_0}{2\vec{q}\cdot\vec{g}}\right)\frac{\partial \psi_1}{\partial t} + \lambda^2(n+1)\psi_1 = 0, \tag{24}$$

and

$$\frac{\partial^2 \psi_2}{\partial t^2} - 2\mathrm{i}\vec{q}\cdot\vec{g}\left(t - \frac{\triangle_0}{2\vec{q}\cdot\vec{g}}\right)\frac{\partial \psi_2}{\partial t} + \lambda^2(n+1)\psi_2 = 0, \tag{25}$$

where

$$\triangle_0(\vec{p}) = \frac{1}{2}\left[\omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\vec{p}}{M} + 3\frac{\hbar q^2}{2M}\right)\right] \tag{26}$$

is time-independent. The exact solutions of equations (24) and (25) read as, respectively,

$$\psi_1(t) = \exp(\mathrm{i}\triangle_1 t)\left(C(1)H(A_n, B_t) + C(2)\,{}_1F_1\left(-A_n, \frac{1}{2}; B_t^2\right)\right), \tag{27}$$

and

$$\psi_2(t) = C(1)H(A_n+1, B_t) + C(2)\,{}_1F_1\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right), \tag{28}$$

where by definition $C(1) \equiv \frac{C_1}{C}$, $C(2) \equiv \frac{C_2}{C}$ with

$$C_1 = \psi_1(0)\,{}_1F_1\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; (-D)^2\right) - \psi_2(0)\,{}_1F_1\left(-A_n, \frac{1}{2}; (-D)^2\right), \tag{29}$$

$$C_2 = \psi_1(0)H(A_n+1, -D) - \psi_2(0)H(A_n, -D), \tag{30}$$

$$C = H(A_n, -D)\,{}_1F_1\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; (-D)^2\right) - H(A_n+1, -D)\,{}_1F_1\left(-A_n, \frac{1}{2}; (-D)^2\right), \tag{31}$$

and H, ${}_1F_1$ denote, respectively, the Hermite and the confluent hypergeometric functions. Furthermore, we have

$$A_n = -(2+\mathrm{i}\beta), \qquad \beta = \frac{\Omega_n(\vec{p}, \vec{g}) - \Delta_0^2}{2\vec{q}\cdot\vec{g}}, \tag{32}$$

and

$$B_t = (\gamma t - \eta)(1+\mathrm{i}), \qquad D = \eta(1+\mathrm{i}), \qquad \gamma = \frac{\sqrt{2}}{2}\vec{q}\cdot\vec{g}, \qquad \eta = \frac{\sqrt{2}\Delta_0}{4\sqrt{\vec{q}\cdot\vec{g}}}. \tag{33}$$

We also define

$$\Omega_n(\vec{p}, \vec{g}) = \sqrt{\Omega_n(\vec{p}, 0)^2 + 2\mathrm{i}\vec{q}\cdot\vec{g}}, \tag{34}$$

with $\Omega_n(\vec{p}, 0)^2 = \lambda^2(n+1) + \triangle_0^2$ as the gravity-dependent Rabi frequency. Here, we should mention that our effective description of the model under consideration involves a certain approximation and that the solution of the Schrödinger equation is not an exact analytical solution of the total model. Namely, the approximation is that of a large detuning between the transition frequency of the atom and the frequency of the field mode, implying that real transitions between the atomic levels are neglected and that only virtual transitions are taken into account.

4. Dynamical properties of the model

In this section, we study the influence of the classical gravity field on the quantum statistical properties of the two-level atom and the quantized radiation field.

4.1. Atomic inversion

An important quantity is the atomic population inversion which is expressed by the expression

$$W(t) = \langle\psi(t)|\hat{\sigma}_z|\psi(t)\rangle. \tag{35}$$

By using the atom–field state $|\psi(t)\rangle$ given by (18) we obtain

$$W(t) = \int \mathrm{d}^3p \sum_{n=0} [|\psi_1|^2 - |\psi_2|^2]. \tag{36}$$

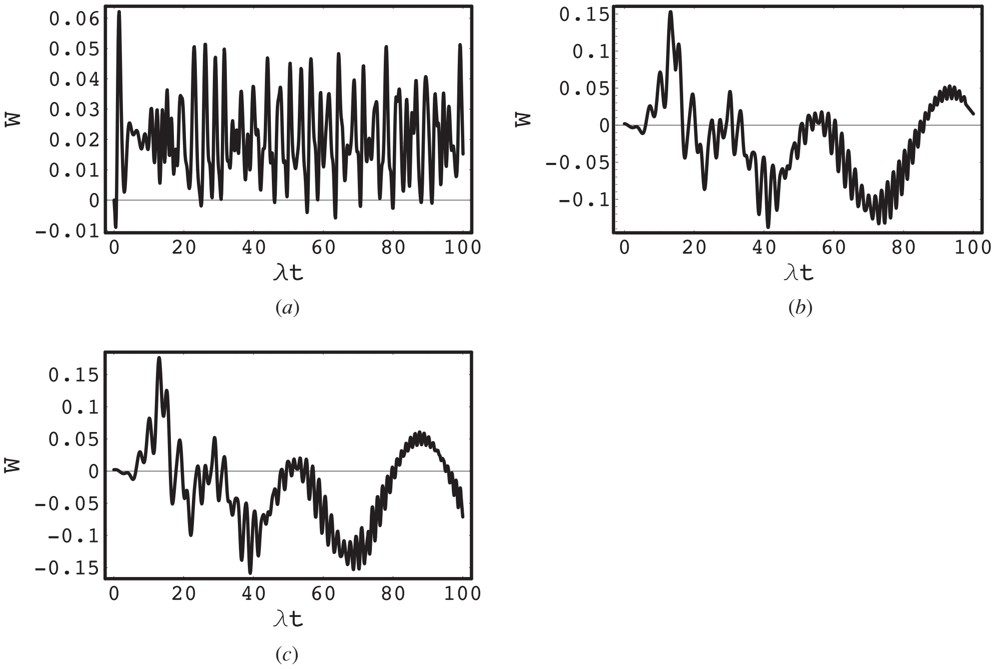


Figure 1. Time evolution of the atomic population inversion versus the scaled time λt. Here we have set $q = 10^7\ \mathrm{m}^{-1}$, $M = 10^{-26}$ kg, $g = 9.8\ \mathrm{m\ s}^{-2}$, $\omega_{\rm rec} = 0.5 \times 10^6\ \mathrm{rad\ s}^{-1}$, $\lambda = 1 \times 10^6\ \mathrm{rad\ s}^{-1}$, $\triangle_0 = 8.5 \times 10^7\ \mathrm{rad\ s}^{-1}$, $\varphi = 0$, $\alpha = 2$, $\Delta = 1.8 \times 10^6\ \mathrm{rad\ s}^{-1}$ and $c_e = c_g = \frac{1}{\sqrt{2}}$ with coherent state for initial cavity field: (*a*) for $\vec{q}\cdot\vec{g} = 0$, (*b*) for $\vec{q}\cdot\vec{g} = 0.5 \times 10^7$, (*c*) for $\vec{q}\cdot\vec{g} = 1.5 \times 10^7$.

Therefore, by substituting from (27) and (28) into (36) we have

$$W(t) = \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \left\{ |C_1|^2 [|H(A_n, B_t)|^2 - |H(A_n + 1, B_t)|^2] + |C_2|^2 \left[\left| {}_1F_1\left(-A_n, \frac{1}{2}; B_t^2\right)\right|^2 \right.\right.$$
$$\left. - \left| {}_1F_1\left(-\frac{1}{2}(A_n+1)\right), \frac{1}{2}; B_t^2\right|^2 \right] + 2\mathrm{Re}\left[C_1 C_2^* \left(H(A_n+1, B_t)\, {}_1F_1^*\left(-A_n, \frac{1}{2}; B_t^2\right) \right.\right.$$
$$\left.\left.\left. - H(A_n+1, B_t)\, {}_1F_1^*\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right)\right)\right]\right\}, \tag{37}$$

where according to (29), (30), (32) and (33), $C(1)$, $C(2)$, A_n and B_t are functions of $\vec{p}$. We assume at $t = 0$, the two-level atom is in a coherent superposition of the excited state and the ground state with $c_g(0) = \frac{1}{\sqrt{2}}$, $c_e(0) = \frac{1}{\sqrt{2}}$. We now consider the influence of gravity on the evolution of atomic population inversion when at $t = 0$, the cavity field is prepared in a Glauber coherent state, $w_n(0) = \frac{\exp(-\frac{|\alpha|^2}{2})\alpha^n}{\sqrt{n!}}$. In figure 1 we have plotted the atomic population inversion as a function of the scaled time λt for three values of the parameter $\vec{q}\cdot\vec{g}$. In this figure and all the subsequent figures we set $q = 10^7\ \mathrm{m}^{-1}$, $M = 10^{-26}$ kg, $g = 9.8\ \mathrm{m\,s}^{-2}$, $\omega_{\rm rec} = \frac{\hbar q^2}{2M} = 0.5 \times 10^6\ \mathrm{rad\ s}^{-1}$, $\lambda = 1 \times 10^6\ \mathrm{rad\ s}^{-1}$, $\triangle_0 = 8.5 \times 10^7\ \mathrm{rad\ s}^{-1}$, $\alpha = 2$, $\Delta = 1.8 \times 10^6\ \mathrm{rad\ s}^{-1}$ and $\phi(\vec{p}) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(\frac{-p^2}{\sigma_0^2}\right)$ with $\sigma_0 = 1$ [28–32]. Here, it is necessary to point out that the relevant time scale introduced by the gravitational influence is $\tau_a = \frac{1}{\sqrt{\vec{q}\cdot\vec{g}}}$ [30]. Therefore for an optical field with $|\vec{q}| = 10^7\ \mathrm{m}^{-1}$, τ_a is about 10^{-4} s. In figure 1(*a*) we consider small gravitational influence. This means very small $\vec{q}\cdot\vec{g}$, i.e., the momentum transfer from the laser beam to the atom is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the laser beam. In figures 1(*b*) and (*c*) we consider the gravitational influence for $\vec{q}\cdot\vec{g} = 0.5\times10^7$ and $\vec{q}\cdot\vec{g} = 1.5\times10^7$, respectively. By comparing figures 1(*a*), (*b*) and (*c*) we can see the influence of gravity on the time evolution of the atomic population inversion. As is seen from figure 1(*a*) for the atomic population inversion the Rabi-like oscillations can be identified. With the increasing value of the parameter $\vec{q}\cdot\vec{g}$ (see figures 1(*b*) and (*c*)) the Rabi oscillations of the atomic population inversion disappear. We calculate the first revival and collapse times and we show that with the increasing value of the parameter $\vec{q}\cdot\vec{g}$, the first revival and collapse times become greater. An estimate of t_c and t_r can be obtained from the conditions

$$(|\Omega_{\langle n\rangle+\sqrt{\langle n\rangle}}| - |\Omega_{\langle n\rangle-\sqrt{\langle n\rangle}}|)t_c \sim 1, \tag{38}$$

and

$$(|\Omega_{\langle n\rangle}| - |\Omega_{\langle n\rangle-1}|)t_r \sim 2m\pi \qquad (m = 1, 2, 3, \ldots), \tag{39}$$

and from (34) we have

$$|\Omega_n| = \left(c_n^2 + d^2\right)^{\frac{1}{4}}, \tag{40}$$

where

$$c_n = \triangle_0^2 + \lambda^2(n+1), \qquad d = 2\vec{q}\cdot\vec{g}. \tag{41}$$

Therefore, we obtain the collapse and revival times as

$$t_c = \frac{1}{(r_{1c} - r_{2c})}, \qquad t_r = \frac{2m\pi}{(r_{1r} - r_{2r})}, \tag{42}$$

where

$$r_{1c} = \left(c^2_{\langle n\rangle+\sqrt{\langle n\rangle}} + d^2\right)^{\frac{1}{4}}, \qquad r_{2c} = \left(c^2_{\langle n\rangle-\sqrt{\langle n\rangle}} + d^2\right)^{\frac{1}{4}}, \tag{43}$$

and

$$r_{1r} = \left(c^2_{\langle n\rangle} + d^2\right)^{\frac{1}{4}}, \qquad r_{2r} = \left(c^2_{\langle n\rangle-1} + d^2\right)^{\frac{1}{4}}. \tag{44}$$

As an example, for the values used in the numerical plots of figure 1 we obtain some of the corresponding first revival and collapse times. We obtain $\lambda t_r = 3.4, 3.5$ and 3.7 and $\lambda t_c = 0.87, 0.88$ and 0.94 for the values $\vec{q}\cdot\vec{g} = 0$, $\vec{q}\cdot\vec{g} = 0.5\times10^7$ and $\vec{q}\cdot\vec{g} = 1.5\times10^7$, respectively. As is seen, with the increasing value of the parameter $\vec{q}\cdot\vec{g}$, the first revival and collapse times become greater.

4.2. Atomic dipole squeezing

To analyse the quantum fluctuations of atomic dipole variables and examine their squeezing we consider the two slowly varying Hermitian quadrature operators

$$\hat{\sigma}_1 = \tfrac{1}{2}(\hat{\sigma}_+ \exp(-\mathrm{i}\omega_{eg}t) + \hat{\sigma}_- \exp(\mathrm{i}\omega_{eg}t)), \tag{45}$$

and

$$\hat{\sigma}_2 = \frac{1}{2\mathrm{i}}(\hat{\sigma}_+ \exp(-\mathrm{i}\omega_{eg}t) - \hat{\sigma}_- \exp(\mathrm{i}\omega_{eg}t)). \tag{46}$$

In fact $\hat{\sigma}_1$ and $\hat{\sigma}_2$ correspond to the dispersive and absorptive components of the amplitude of the atomic polarization [2], respectively. They obey the commutation relation $[\hat{\sigma}_1, \hat{\sigma}_2] = \frac{\mathrm{i}}{2}\hat{\sigma}_3$. Correspondingly, the Heisenberg uncertainty relation is

$$(\Delta\hat{\sigma}_1)^2(\Delta\hat{\sigma}_2)^2 \geqslant \tfrac{1}{16}|\langle\hat{\sigma}_3\rangle|^2, \tag{47}$$

where $(\Delta\hat{\sigma}_i)^2 = \left\langle\hat{\sigma}_i^2\right\rangle - \langle\hat{\sigma}_i\rangle^2$ is the variance in the component $\hat{\sigma}_i$ $(i = 1, 2)$ of the atomic dipole.

The fluctuations in the component $\hat{\sigma}_i$ $(i = 1, 2)$ are said to be squeezed (i.e., dipole squeezing) if the variance in $\hat{\sigma}_i$ satisfies the condition

$$(\Delta\hat{\sigma}_i)^2 < \tfrac{1}{4}|\langle\hat{\sigma}_3\rangle|, \qquad (i = 1 \text{ or } 2). \tag{48}$$

Since $\hat{\sigma}_i^2 = \frac{1}{4}$ this condition may be written as

$$F_i = 1 - 4\langle\hat{\sigma}_i\rangle^2 - |\langle\hat{\sigma}_3\rangle| < 0, \qquad (i = 1 \text{ or } 2). \tag{49}$$

The expectation values of the atomic operators $\hat{\sigma}_1$ and $\hat{\sigma}_2$ in the state $|\psi(t)\rangle$ of the atom–field system, given by (18), are

$$\langle\hat{\sigma}_1\rangle = \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \mathrm{Re}[\psi_1(t)\psi_2^*(t)\exp(-\mathrm{i}\omega_{eg}t)], \tag{50}$$

and

$$\langle\hat{\sigma}_2\rangle = \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \mathrm{Im}[\psi_1(t)\psi_2^*(t)\exp(-\mathrm{i}\omega_{eg}t)]. \tag{51}$$

Therefore, by substituting from (27) and (28) into (50) and (51) we have

$$\begin{aligned}\langle\hat{\sigma}_1\rangle = \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \mathrm{Re}\Bigg[\Bigg(&\exp(\mathrm{i}\triangle_1 t)\Bigg(C(1)H(A_n, B_t)\\ &+C(2)\,{}_1F_1\left(-A_n, \frac{1}{2}; B_t^2\right)\Bigg)\Bigg)\Bigg(C^*(1)H^*(A_n+1, B_t)\\ &+C^*(2)\,{}_1F_1^*\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right)\Bigg)\exp(-\mathrm{i}\omega_{eg}t)\Bigg],\end{aligned} \tag{52}$$

and

$$\begin{aligned}\langle\hat{\sigma}_2\rangle = \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \mathrm{Im}\Bigg[\Bigg(&\exp(\mathrm{i}\triangle_1 t)\Bigg(C(1)H(A_n, B_t)\\ &+C(2)\,{}_1F_1\left(-A_n, \frac{1}{2}; B_t^2\right)\Bigg)\Bigg)\Bigg(C^*(1)H^*(A_n+1, B_t)\\ &+C^*(2)\,{}_1F_1^*\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right)\Bigg)\exp(-\mathrm{i}\omega_{eg}t)\Bigg].\end{aligned} \tag{53}$$

The time evolution of $F_1(t)$ corresponding to the squeezing of $\hat{\sigma}_1$ has been shown in figure 2 for three values of the parameter $\vec{q}\cdot\vec{g}$. As it is seen, with the increasing value of the parameter $\vec{q}\cdot\vec{g}$ the dipole squeezing is completely removed.

4.3. Atomic momentum diffusion

The next quantity we consider is the atomic momentum diffusion. As a consequence of the atomic momentum diffusion, the atom experiences light-induced forces (radiation force) during its interaction with the radiation field. The atomic momentum diffusion is given by

$$\Delta p(t) = (\langle\hat{p}(t)^2\rangle - \langle\hat{p}(t)\rangle^2)^{\frac{1}{2}}. \tag{54}$$

By using (18) and $\hat{p}|p\rangle = p|p\rangle$, we obtain

$$\Delta p(t) = \left\{\left[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p\, p^2(|\psi_1|^2 + |\psi_2|^2)\right] - \left[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p\, p(|\psi_1|^2 + |\psi_2|^2)\right]^2\right\}^{\frac{1}{2}}. \tag{55}$$

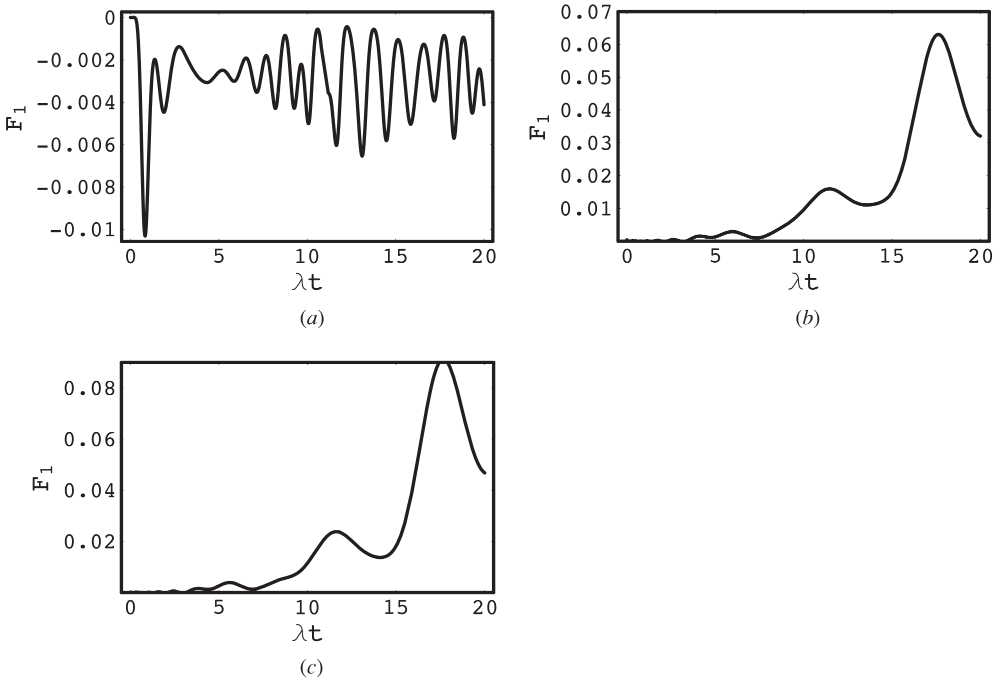
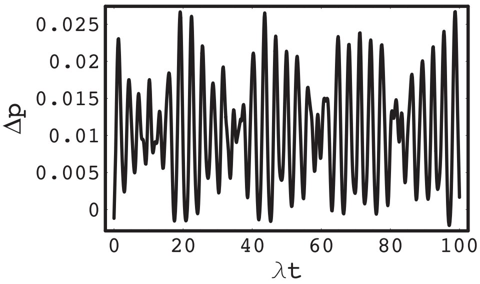


Figure 2. Time evolution of the atomic dipole squeezing versus the scaled time λt with the same corresponding data used in figure 1: (a) for $\vec{q}\cdot\vec{g}=0$, (b) for $\vec{q}\cdot\vec{g}=0.5\times10^7$, ($c$) for $\vec{q}\cdot\vec{g}=1.5\times10^7$.

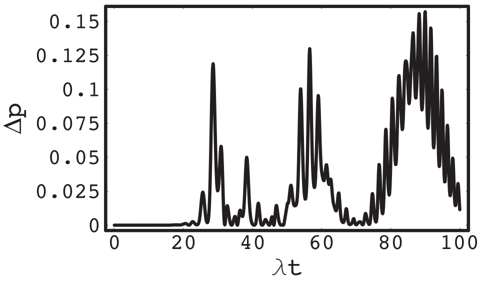
By substituting (27) and (28) into (55) we obtain

$$\begin{aligned}\Delta p(t)=\Bigg\{\Bigg[\sum_{n=0}^{\infty}\int \mathrm{d}^3p\,p^2\Bigg(\Bigg\{&|C_1|^2[|H(A_n,B_t)|^2+|H(A_n+1,B_t)|^2]\\&+|C_2|^2\Bigg[\left|{}_1F_1\left(-A_n,\frac{1}{2};B_t^2\right)\right|^2+\left|{}_1F_1\left(-\frac{1}{2}(A_n+1),\frac{1}{2};B_t^2\right|^2\right]\\&+2\mathrm{Re}\Bigg[C_1C_2^*\Bigg(H(A_n+1,B_t)\,{}_1F_1^*\left(-A_n,\frac{1}{2};B_t^2\right)+H(A_n+1,B_t)\\&\times{}_1F_1^*\left(-\frac{1}{2}(A_n+1),\frac{1}{2};B_t^2\right)\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg]-\Bigg[\sum_{n=0}^{\infty}\int\mathrm{d}^3p\,p\Bigg(\Bigg\{|C_1|^2\Bigg[|H(A_n,B_t)|^2\\&+|H(A_n+1,B_t)|^2\Bigg]+|C_2|^2\Bigg[\left|{}_1F_1\left(-A_n,\frac{1}{2};B_t^2\right)\right|^2\\&+\left|{}_1F_1\left(-\frac{1}{2}(A_n+1),\frac{1}{2};B_t^2\right|^2\right]+2\mathrm{Re}\Bigg[C_1C_2^*(H(A_n+1,B_t)\\&\times{}_1F_1^*\left(-A_n,\frac{1}{2};B_t^2\right)+H(A_n+1,B_t)\,{}_1F_1^*\left(-\frac{1}{2}(A_n+1),\frac{1}{2};B_t^2\right)\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg]^2\Bigg\}^{\frac{1}{2}}.\end{aligned}\tag{56}$$

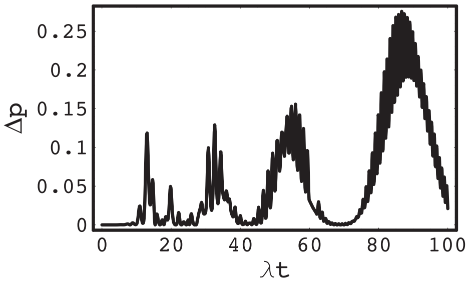
In figure 3 we have plotted $\Delta p(t)$ for $\vec{q}\cdot\vec{g}=0$, $\vec{q}\cdot\vec{g}=0.5\times10^7$ and $\vec{q}\cdot\vec{g}=1.5\times10^7$, respectively. In figure 3(a) the Rabi-like oscillations can be identified, but in figures 3(b)



(*a*)



(*b*)



(*c*)

Figure 3. Time evolution of the atomic momentum diffusion versus the scaled time λt with the same corresponding data used in figure 1: (*a*) for $\vec{q}\cdot\vec{g} = 0$, (*b*) for $\vec{q}\cdot\vec{g} = 0.5\times 10^7$, (*c*) for $\vec{q}\cdot\vec{g} = 1.5\times 10^7$.

and (*c*), when the influence of the gravitational field increases, the Rabi oscillations disappear. Moreover, the atom can experience larger light-induced forces during its interaction with the radiation field, when the gravitational field increases.

4.4. The probability distribution of the cavity field

The probability distribution function $P(n, t)$ that there are n photons in the cavity field at time t is given by

$$P(n,t) = |\langle n|\psi(t)\rangle|^2. \tag{57}$$

By using the expressions (18), (27) and (28) we have

$$P(n,t) = \int \mathrm{d}^3 p[|\psi_1(t)|^2 + |\psi_2(t)|^2]. \tag{58}$$

Therefore, we obtain

$$\begin{aligned} P(n,t) = \int \mathrm{d}^3 p \bigg\{ & |C_1|^2[|H(A_n, B_t)|^2 + |H(A_n+1, B_t)|^2] \\ & + |C_2|^2 \left[\left| {}_1F_1\left(-A_n, \frac{1}{2}; B_t^2\right)\right|^2 + \left|{}_1F_1\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right|^2 \right] \\ & + 2\mathrm{Re}\left[C_1 C_2^* \left(H(A_n+1, B_t)\, {}_1F_1^*\left(-A_n, \frac{1}{2}; B_t^2\right) \right.\right. \\ & \left.\left. + H(A_n+1, B_t)\, {}_1F_1^*\left(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\right)\right)\right]\bigg\}. \end{aligned} \tag{59}$$

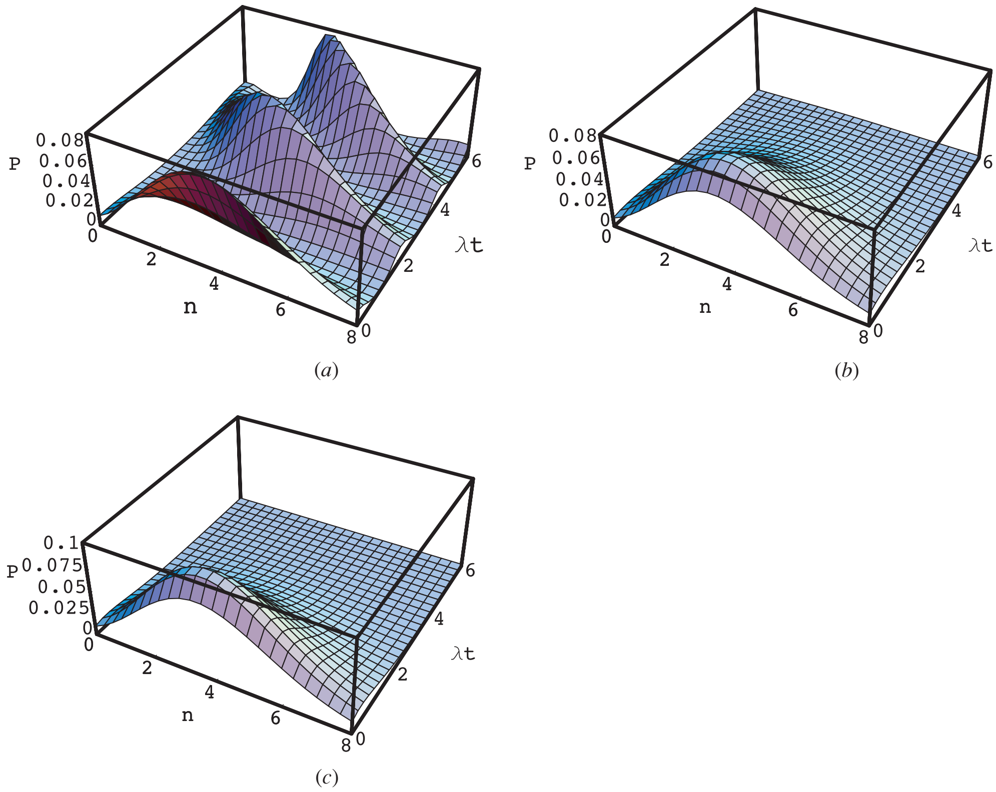


Figure 4. The three-dimensional plot of the probability distribution function $P(n, t)$ versus the scaled time λt and n with the same corresponding data used in figure 1: (a) for $\vec{q}\cdot\vec{g} = 0$, (b) for $\vec{q}\cdot\vec{g} = 0.5\times 10^7$, (c) for $\vec{q}\cdot\vec{g} = 1.5\times 10^7$.

In figure 4 we have shown the three-dimensional plot of the probability distribution of the cavity field $P(n, t)$ with the same corresponding data used in figure 1 with $\alpha = 2$. By comparing figures 4(a), (b) and (c) we can see that the multi-peak structure of $P(n, t)$ is destroyed with increase of the parameter $\vec{q}\cdot\vec{g}$. This result can be considered as a first evidence for the suppression of nonclassical behaviour of the cavity field in the presence of gravity.

4.5. Photon counting statistics

We now investigate the influence of gravity on the sub-Poissonian statistics of the radiation field. For this purpose, we calculate the Mandel parameter defined by [33]

$$Q(t) = \frac{(\langle n(t)^2\rangle - \langle n(t)\rangle^2)}{\langle n(t)\rangle} - 1. \tag{60}$$

For $Q < 0$ ($Q > 0$), the statistics is sub-Poissonian (super-Poissonian); $Q = 0$ stands for Poissonian statistic. Since $\langle n(t)\rangle = \sum_{n=0}^{\infty} nP(n, t)$ and $\langle n(t)^2\rangle = \sum_{n=0}^{\infty} n^2 P(n, t)$ we have

$$Q(t) = \left(\left\{\left[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n^2 P(n, t)\right] - \left[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n P(n, t)\right]^2\right\}\left[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n P(n, t)\right]^{-1}\right) - 1. \tag{61}$$

Therefore, by using (59), (60) and (61) we obtain

$$
\begin{aligned}
Q(t) = \Bigg(\Bigg\{\Bigg[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n^2 \Bigg(\Bigg\{ & |C_1|^2[|H(A_n, B_t)|^2 + |H(A_n+1, B_t)|^2] \\
& + |C_2|^2\Bigg[\Bigg|{}_1F_1\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg)\Bigg|^2 + \Bigg|{}_1F_1\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg|^2\Bigg] \\
& + 2\mathrm{Re}\Bigg[C_1C_2^*(H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg) \\
& + H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg)\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg] \\
& - \Bigg[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n\Bigg(\Bigg\{|C_1|^2[|H(A_n, B_t)|^2 + |H(A_n+1, B_t)|^2] \\
& + |C_2|^2\Bigg[\Bigg|{}_1F_1\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg)\Bigg|^2 + \Bigg|{}_1F_1\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg|^2\Bigg] \\
& + 2\mathrm{Re}\Bigg[C_1C_2^*\Bigg(H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg) \\
& + H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg)\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg]^2\Bigg\} \\
& - \Bigg[\sum_{n=0}^{\infty}\int \mathrm{d}^3 p n\Bigg(\Bigg\{|C_1|^2[|H(A_n, B_t)|^2 + |H(A_n+1, B_t)|^2] \\
& + |C_2|^2\Bigg[\Bigg|{}_1F_1\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg)\Bigg|^2 + \Bigg|{}_1F_1\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg|^2\Bigg] \\
& + 2\mathrm{Re}\Bigg[C_1C_2^*(H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-A_n, \frac{1}{2}; B_t^2\Bigg) \\
& + H(A_n+1, B_t)\,{}_1F_1^*\Bigg(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2\Bigg)\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg]^{-1}\Bigg) - 1. \qquad (62)
\end{aligned}
$$

The numerical results for three values of the parameter $\vec{q}\cdot\vec{g}$ are shown in figure 5 for the Mandel parameter. As it is seen, the field exhibits alternately sub-Poissonian and super-Poissonian statistics when the influence of the gravitational field is negligible. With increasing $\vec{q}\cdot\vec{g}$ the sub-Poissonian characteristic is suppressed and the cavity field exhibits super-Poissonian statistics.

4.6. Quadrature squeezing of the cavity field

Finally, we investigate the influence of gravity on the quadrature squeezing of the radiation field. For this purpose, we introduce two slowly varying quadrature operators

$$\hat{X}_1(t) = \tfrac{1}{2}(\hat{a}\exp(\mathrm{i}\omega t) + \hat{a}^\dagger\exp(-\mathrm{i}\omega t)), \qquad (63)$$

and

$$\hat{X}_2(t) = \frac{1}{2\mathrm{i}}(\hat{a}\exp(\mathrm{i}\omega t) - \hat{a}^\dagger(-\mathrm{i}\omega t)), \qquad (64)$$

where $\hat{a}$ and $\hat{a}^\dagger$ obey the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. The operators $\hat{X}_1(t)$ and $\hat{X}_2(t)$ satisfy the commutation relation

$$[\hat{X}_1(t), \hat{X}_2(t)] = \frac{\mathrm{i}}{2}, \qquad (65)$$

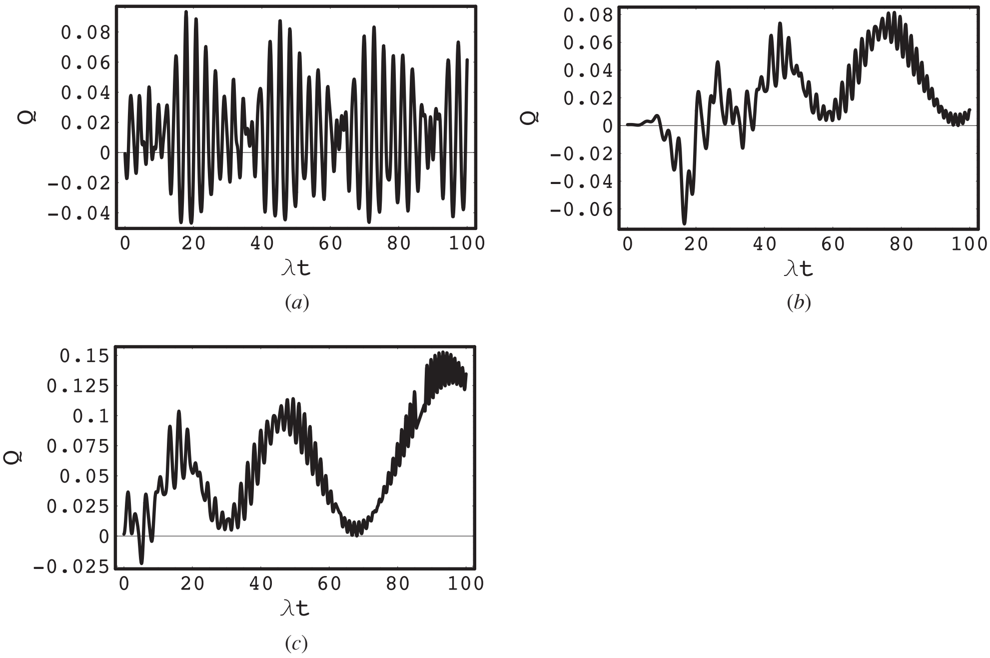


Figure 5. Time evolution of the Mandel parameter $Q(t)$ versus the scaled time λt with the same corresponding data used in figure 1: (*a*) for $\vec{q}\cdot\vec{g}=0$, (*b*) for $\vec{q}\cdot\vec{g}=0.5\times10^{7}$, (*c*) for $\vec{q}\cdot\vec{g}=1.5\times10^{7}$.

which implies the Heisenberg uncertainty relation

$$\langle(\triangle\hat{X}_1(t))^2\rangle\langle(\triangle\hat{X}_2(t))^2\rangle \geqslant \tfrac{1}{16}. \tag{66}$$

A state of the radiation field is said to be squeezed whenever

$$\langle(\triangle\hat{X}_i)^2\rangle < \tfrac{1}{4}, \qquad (i=1\,\text{or}\,2), \tag{67}$$

where

$$\langle(\triangle\hat{X}_i)^2\rangle = \left\langle\hat{X}_i^2\right\rangle - \left\langle\hat{X}_i\right\rangle^2, \qquad (i=1,2). \tag{68}$$

The degree of squeezing can be measured by the squeezing parameter S_i, $(i=1,2)$ defined by

$$S_i(t) = 4\langle(\triangle\hat{X}_i(t))^2\rangle - 1, \tag{69}$$

which can be expressed in terms of the annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$ as follows

$$\begin{aligned} S_1(t) = {} & (\langle\hat{a}^2(t)\rangle - \langle\hat{a}(t)\rangle^2)\exp(2\mathrm{i}\omega t) + (\langle\hat{a}^{\dagger 2}(t)\rangle - \langle\hat{a}^{\dagger}(t)\rangle^2) \\ & \times\exp(-2\mathrm{i}\omega t) + 2(\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \langle\hat{a}^\dagger(t)\rangle\langle\hat{a}(t)\rangle), \end{aligned} \tag{70}$$

and

$$\begin{aligned} S_2(t) = {} & -(\langle\hat{a}^2(t)\rangle - \langle\hat{a}(t)\rangle^2)\exp(2\mathrm{i}\omega t) - (\langle\hat{a}^{\dagger 2}(\vec{p},t)\rangle - \langle\hat{a}^{\dagger}(t)\rangle^2) \\ & \times\exp(-2\mathrm{i}\omega t) + 2(\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \langle\hat{a}^\dagger(t)\rangle\langle\hat{a}(t)\rangle). \end{aligned} \tag{71}$$

Then, the condition for squeezing in the quadrature component can be simply written as $S_i(t) < 0$. By using (18), (27) and (28) we obtain

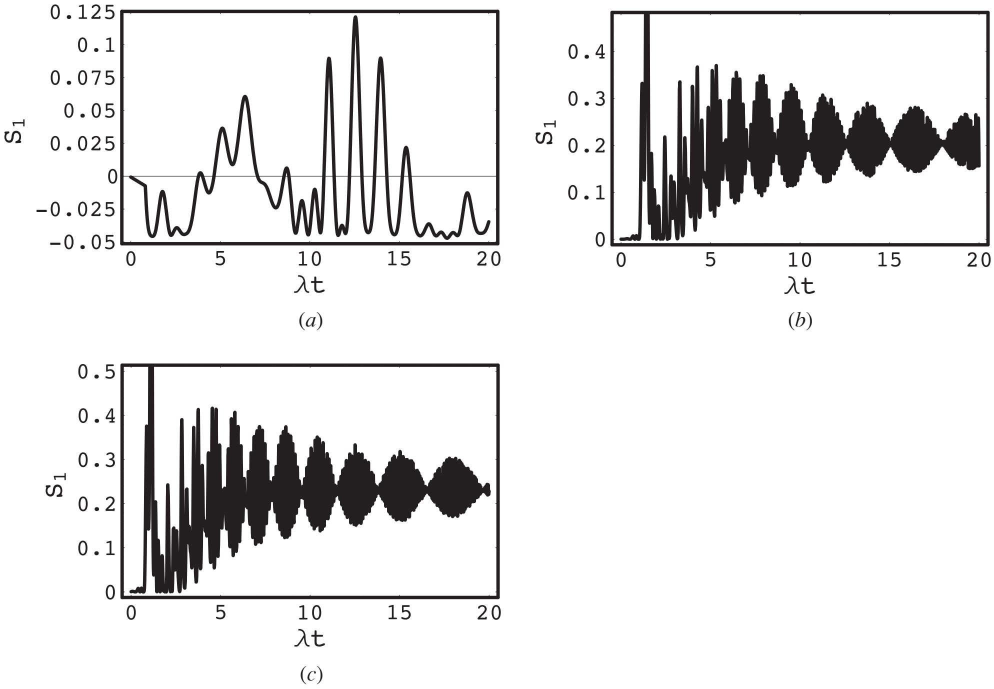


Figure 6. Time evolution of the squeezing parameter $S_1(t)$ versus the scaled time λt with the same corresponding data used in figure 1: (*a*) for $\vec{q}\cdot\vec{g} = 0$, (*b*) for $\vec{q}\cdot\vec{g} = 0.5\times10^7$, (*c*) for $\vec{q}\cdot\vec{g} = 1.5\times10^7$.

$$\langle\hat{a}(t)\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}^3p(\sqrt{n}\psi_{1n}\psi^*_{1(n-1)} + \sqrt{n+1}\psi_{2n}\psi^*_{2(n-1)}), \tag{72}$$

$$\langle\hat{a}^\dagger(t)\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}^3p(\sqrt{n+1}\psi_{1n}\psi^*_{1(n+1)} + \sqrt{n+2}\psi_{2n}\psi^*_{2(n+1)}), \tag{73}$$

$$\langle\hat{a}^2(t)\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}^3p(\sqrt{n(n-1)}\psi_{1n}\psi^*_{1(n-2)} + \sqrt{n(n+1)}\psi_{2n}\psi^*_{2(n-2)}), \tag{74}$$

$$\langle\hat{a}^{\dagger2}(t)\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}^3p(\sqrt{(n+1)(n+2)}\psi_{1n}\psi^*_{1(n+2)} + \sqrt{(n+2)(n+3)}\psi_{2n}\psi^*_{2(n+3)}), \tag{75}$$

with

$$\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}^3p(n\psi_{1n}\psi^*_{1n} + (n+1)\psi_{2n}\psi^*_{2(n-1)}). \tag{76}$$

In figure 6 we have plotted the squeezing parameter S_1 versus the scaled time λt for three values of the parameter $\vec{q}\cdot\vec{g}$. As it is seen, the quadrature component $\hat{X}_1$ exhibits squeezing in the course of time evolution when the influence of the gravitational field is negligible. With increase of the parameter $\vec{q}\cdot\vec{g}$, the parameter S_1 shows fast oscillatory behaviour and the quadrature squeezing decays.

5. Summary and conclusions

We studied the temporal evolution of quantum statistical properties of an interacting atom–radiation system in the presence of a classical homogeneous gravitational field within the framework of the Jaynes–Cummings model. According to the equivalence principle the influence of a homogeneous classical gravity field on the atom moving in a radiation field can be simulated by constant acceleration. This means that our system is physically equivalent to the situation where a two-level atom is at rest or moving with constant velocity relative to an inertial system and the laboratory with the radiation field attached to it moves with constant acceleration. To analyse the dynamical evolution of the atom–radiation system, we presented a quantum treatment of the internal and external dynamics of the atom based on an alternative su(2) dynamical algebraic structure. By solving the Schrödinger equation in the interaction picture, we found the evolving state of the system by which the influence of the classical gravity field on the dynamical behaviour of the atom–radiation system was explored. Assuming that initially the radiation field has been prepared in a coherent state and the two-level atom has been prepared in a coherent superposition of the excited and ground states, we discussed the influence of gravity on the collapses and revivals of the atomic population inversion, atomic dipole squeezing, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field. The results are summarized as follows: (1) the Rabi-like oscillations in the atomic population inversion disappear with increase of the gravitational field influence, (2) the dipole squeezing decays with increase of the parameter $\vec{q} \cdot \vec{g}$, (3) in the presence of the gravitational field, the atom can experience larger light-induced forces during its interaction with the radiation field, (4) the multi-peak structure of the photon-number distribution is destroyed with increase of the gravitational field influence, (5) with increasing $\vec{q} \cdot \vec{g}$, the sub-Poissonian behaviour of the cavity field is suppressed and it exhibits super-Poissonian statistics and (6) the quadrature squeezing of the cavity field decays with increase of the parameter $\vec{q} \cdot \vec{g}$.

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References

[1] Jaynes E T and Cummings F 1963 *Proc. IEEE* **51** 89
[2] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 *Phys. Rev. Lett.* **44** 1323
Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 *Phys. Rev.* A **23** 236
Yoo H I, Sanchez-Mondragon J J and Eberly J H 1981 *J. Phys. A: Math. Gen* **14** 1383
[3] Kuklinski J R and Madajczyk J 1988 *Phys. Rev.* A **37** 3175
Woods C W and Gea-Banacloche J 1993 *J. Mod. Opt.* **40** 2361
Kozierowski M, Poyatos J F and Sanches-Soto L L 1995 *J. Mod. Opt.* **42** 569
[4] Barnett S M 1982 *Opt. Commun.* **61** 432
Barnett S M and Knight P L 1988 *Phys. Scr.* T **21** 5
Li X S, Lin D L, George T F and Liu Z D 1989 *Phys. Rev.* A **40** 228
Zhou P and Peng J S 1991 *Phys. Rev.* A **44** 3331
[5] Agarwal G S 1985 *J. Opt. Soc. Am.* B **2** 480
[6] Phoenix S J D and Knight P L 1988 *Ann. Phys., NY* **186** 381
[7] Phoenix S J D and Knight P L 1991 *Phys. Rev.* A **44** 6023
[8] Ekert A and Knight P L 1995 *Am. J. Phys.* **63** 415
[9] Mecshede D, Walther H and Muller G 1985 *Phys. Rev. Lett.* **54** 551

[10] Kyungwon An, Childs J J, Dasari R R and Feld M S 1994 *Phys. Rev. Lett.* **73** 3375
[11] Meekhof D M, Monroe C, King B E, Itano W M and Wineland D J 1996 *Phys. Rev. Lett.* **76** 1796
[12] Fleischhauer M and Schleich W P 1993 *Phys. Rev.* A **47** 4258
D'Helen C and Milburn G J 1995 *Phys. Rev.* A **52** 4755
Munro W J, Nemoto Kao, Beau Soleil R G and Spiller T P 2005 *Phys. Rev.* A **71** 033819
[13] Sukumar C V and Buck B 1981 *Phys. Lett.* A **83** 211
Yoo Y I and Eberly J H 1985 *Phys. Rep.* **118** 239
Joshi A and Puri R R 1987 *J. Mod. Opt.* **34** 1421
Shore B W and Knight P L 1989 *J. Mod. Opt.* **40** 137
Chough Y T and Charmichael H J 1996 *Phys. Rev.* A **54** 1709
Joshi A 2000 *Phys. Rev.* A **62** 043812
Zait R A and El-Wahab N H Abd 2002 *J. Phys. B: At. Mol. Opt. Phys.* **35** 3701
Zait R A 2003 *Phys. Lett.* A **319** 461
[14] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J and Haroche S 2000 *Science* **288** 2024
Soronsen A S and Molmer K 2003 *Phys. Rev. Lett.* **91** 097905
[15] Schicher R R 1989 *Opt. Commun.* **70** 97
[16] Joshi A and Lawande S V 1990 *Phys. Rev.* A **42** 1752
[17] Joshi A and Lawande S V 1992 *Int. J. Mod. Phys.* B **6** 3539
[18] Bartzisl V 1992 *Physica* A **180** 428
[19] Bimalendu D and Surajit S 1997 *Phys. Rev.* A **56** 2470
[20] Meyer G M, Scully M O and Walther H 1997 *Phys. Rev.* A **56** 4142
[21] Fang Mao-Fa 1998 *Physica* A **259** 193
[22] Joshi A 1998 *Phys. Rev.* A **58** 4662
[23] Liaoa Xiang-Ping and Fang Mao-Fa 2004 *Physica* A **332** 176
[24] Joshi A and Xiao Min 2004 *Opt. Commun.* **232** 273
[25] Schlicher R R 1989 *Opt. Commun.* **70** 97
Joshi A and Xiao Min 2004 *J. Opt. Soc. Am.* B **21** 1685
[26] Adamas C, Sigel M and Mlynek J 1994 *Phys. Rep.* **240** 143
[27] Kastberg A, Philips W D, Rolston S L, Spreeuw R J C and Jessen P S 1995 *Phys. Rev. Lett.* **74** 1542
[28] Lammerzahl C and Borde C J 1995 *Phys. Lett.* A **203** 59
[29] Marzlin K P and Audertsch J 1995 *Phys. Rev.* A **53** 1004
[30] Mohammadi M, Naderi M H and Soltanolkotabi M 2006 *J. Phys. A: Math. Gen.* **39** 11065
[31] Sleator T and Wilkens M 1993 *Phys. Rev.* A **48** 3286
[32] de Matos Filho R L and Vogel W 1996 *Phys. Rev. Lett.* **76** 608
[33] Mandel L 1979 *Opt. Lett.* **4** 205
Mandel L 1986 *Phys. Scr.* **72** 34